Tepper Gill, Kexi Liu, and Eric Trell, Editors
Fundamental Open Problems in Science at the End of the Millennium
Proceedings of the Beijing Workshop, August 1997
Hadronic Press, Palm Harbor, FL 34682-1577, U.S.A.
ISBN 1-57485-029-6, pp. 555-558

Fermat Last Theorem was Proved in 1991

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We found out a new method for proving Fermat last theorem (FLT) on the afternoon of October 25, 1991. We proved FLT at one stroke for all prime exponents $p>3$, It led to the discovery to calculate $n=15, 21, 35, 105, \cdots\cdots$. To this date, no one disprove this proof. Anyone can not deny it, because it is a simple and marvelous proof. It can fit in the margin of Fermat book.

In 1974 we found out Euler formula of the cyclotomic real numbers in the cyclotomic fields [1].

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right)=\sum_{i=1}^{n} S_i J^{i-1} \tag{1}$$

where J denotes a nth root of unity, $J^n=1$, n is an odd number, t_i are the real numbers.

S_i is called thd complex hyperbolic functions of order n with $n-1$ variables,

$$S_i=\frac{1}{n}[e^A+2\sum_{j=1}^{\frac{n-1}{2}}(-1)^{(i-1)j}e^{B_j}\cos(\theta_j+(-1)^j\frac{(i-1)j\pi}{n})] \tag{2}$$

where

$$A=\sum_{\alpha=1}^{n-1}t_\alpha,\quad B_j=\sum_{\alpha=1}^{n-1}t_\alpha(-1)^{\alpha j}\cos\frac{\alpha j\pi}{n},\quad \theta_j=(-1)^{j+1}\sum_{\alpha=1}^{n-1}t_\alpha(-1)^{\alpha j}\sin\frac{\alpha j\pi}{n},$$

$$A+2\sum_{j=1}^{\frac{n-1}{2}}B_j=0 \tag{3}$$

Using (1) the cyclotomic theory may extend to totally real number fields. It is called the hypercomplex variable theory [1]. (2) may be written in the matrix form

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix}=\frac{1}{n}\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} \tag{4}$$

where $(n-1)/2$ is an even number.

From (4) we may obtain its inverse transformation

$$\begin{bmatrix} e^{A} \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n} \\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdots \\ S_n \end{bmatrix} \quad (5)$$

From (5) we have

$$e^{A} = \sum_{i=1}^{n} S_i, \quad e^{B_j}\cos\theta_j = S_1 + \sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\cos\frac{ij\pi}{n},$$

$$e^{B_j}\sin\theta_j = (-1)^{j+1}\sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\sin\frac{ij\pi}{n}, \quad (6)$$

In (3) and (6) t_i and S_i have the same formulas such that every factor of n has a Fermat equation. Assume $S_1 \neq 0$, $S_2 \neq 0$, $S_i = 0$ where $i = 3, 4, \cdots\cdots, n$. $S_i = 0$ are $n-2$ indeterminate equations with $n-1$ variables. From (6) we have

$$e^{A} = S_1 + S_2, \quad e^{2B_j} = S_1^2 + S_2^2 + 2S_1S_2(-1)^j\cos\frac{j\pi}{n}. \quad (7)$$

From (3) and (7) we may obtain Fermat equation

$$\exp(A + 2\sum_{j=1}^{\frac{n-1}{2}} B_j) = (S_1 + S_2)\prod_{j=1}^{\frac{n-1}{2}}(S_1^2 + S_2^2 + 2S_1S_2(-1)^j\cos\frac{j\pi}{n}) = S_1^n + S_2^n = 1 \quad (8)$$

Theorem. Fermat last theorem has no rational solutions with $S_1S_2 \neq 0$ for all odd exponents.

Proof. The proof of FLT is difficult when n is an odd prime. We consider n is a composite number.

Let $n = \prod n_i$, where n_i ranges over all odd numbers. From (3) we have

$$\exp(A + 2\sum_{j=1}^{\frac{f-1}{2}} B_{\frac{n}{f}j}) = [\exp(\sum_{\alpha=1}^{\frac{n}{f}-1} t_{f\alpha})]^f \quad (9)$$

From (7) we have

$$\exp(A + 2\sum_{j=1}^{\frac{f-1}{2}} B_{\frac{n}{f}j}) = S_1^f + S_2^f \quad (10)$$

where f is a factor of n. From (9) and (10) we may obtain Fermat equation

$$\exp(A + 2\sum_{j=1}^{\frac{f-1}{2}} B_{\frac{n}{f}j}) = S_1^f + S_2^f = [\exp(\sum_{\alpha=1}^{\frac{n}{f}-1} t_{f\alpha})]^f \quad (11)$$

Every factor of n has a Fermat equation. From (11) we have

$$f = 1, \quad B_n = B_0 = 0, \quad e^{A} = S_1 + S_2 = \exp(\sum_{\alpha=1}^{n-1} t_\alpha) \quad (12)$$

$$f=n,\quad t_n=t_0=0,\quad \exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=S_1^n+S_2^n=1 \tag{13}$$

$$f=3,\quad \exp(A+2B_{\frac{n}{3}})=S_1^3+S_2^3=[\exp(\sum_{\alpha=1}^{\frac{n}{3}-1}t_{3\alpha})]^3 \tag{14}$$

If $S_1=1$, $S_2=0$ and $S_1=0$, $S_2=1$, then $A=B_j\ =0$. Euler proved (13), therefore (11) has no rational solutions with $S_1S_2\neq0$ (and so no integer solutions with $S_1S_2\neq0$) for all odd exponents f. (11) and (13) can fit in the margin of Fermat book.

Let $n=3p$ where p is an odd prime. From (3) and (7) we may derive Fermat equations

$$\exp(A+2\sum_{j=1}^{\frac{3p-1}{2}}B_j)=S_1^{3p}+S_2^{3p}=(S_1^p)^3+(S_2^p)^3=1 \tag{15}$$

$$\exp(A+2B_p)=S_1^3+S_2^3=[\exp(\sum_{\alpha=1}^{p-1}t_{3\alpha})]^3 \tag{16}$$

$$\exp(A+2\sum_{j=1}^{\frac{p-1}{2}}B_{3j})=S_1^p+S_2^p=[\exp(t_p+t_{2p})]^p \tag{17}$$

Euler proved (15), therefore (16) and (17) have no rational solutions with $S_1S_2\neq0$ (and so no integer solutions with $S_1S_2\neq0$) for any odd prime $p>3$. (15)—(17) can fit in the margin.

Let $n=5p$, where p is an odd prime. From (3) and (7) we may derive Fermat equations

$$\exp(A+2\sum_{j=1}^{\frac{5p-1}{2}}B_j)=S_1^{5p}+S_2^{5p}=1 \tag{18}$$

$$\exp(A+2B_p+2B_{2p})=S_1^5+S_2^5=[\exp(\sum_{\alpha=1}^{p-1}t_{5\alpha})]^5 \tag{19}$$

$$\exp(A+2\sum_{j=1}^{\frac{p-1}{2}}B_{5j})=S_1^p+S_2^p=[\exp(\sum_{\alpha=1}^{4}t_{p\alpha})]^p \tag{20}$$

(18)—(20) can fit in the margin.

Let $n=7p$ where p is an odd prime. From (3) and (7) we may derive Fermat equations

$$\exp(A+2\sum_{j=1}^{\frac{7p-1}{2}}B_j)=S_1^{7p}+S_2^{7p}=1 \tag{21}$$

$$\exp(A+2B_p+2B_{2p}+2B_{3p})=S_1^7+S_2^7=[\exp(\sum_{\alpha=1}^{p-1}t_{7\alpha})]^7 \tag{22}$$

$$\exp(A+2\sum_{j=1}^{\frac{p-1}{2}}B_{7j})=S_1^p+S_2^p=[\exp(\sum_{\alpha=1}^{6}t_{p\alpha})]^p \tag{23}$$

(21)—(23) can also fit in the margin.

Using this method we proved FLT in 1991 [2–5].

Let $n=p$ where p is an odd prime. From (3) and (7) we have

$$\exp(A + 2\sum_{j=1}^{\frac{p-1}{2}} B_j) = S_1^p + S_2^p = 1, e^{2B_1} = S_1^2 + S_2^2 - 2S_1S_2\cos\frac{\pi}{p} \tag{24}$$

Let $a = S_1 e^{-B_1}$ and $b = S_2 e^{-B_1}$ From (24) we have

$$a^p + b^p = (e^{-B_1})^p \tag{25}$$

$$a^2 + b^2 - 2ab\cos\frac{\pi}{p} = 1 \tag{26}$$

The proof of (25) is transformed into studying (26). (26) has no rational solutions with $ab \neq 0$, because $\cos\frac{\pi}{p}$ is an irrational number for $p > 3$. Therefore (25) has no rational solutions for any odd prime $p > 3$. (25) and (26) can also fit in the margin.

Remark. If $S_i \neq 0$, where $i = 1,2,3,\cdots,n$,

then (11)—(23) have infinitely many rational solutions [1].

References

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Note. Let one knew the important results, we gave out about 600 preprints in 1991—1992. There were my preprints in Princeton, Harvard, Berkeley, MIT, Chicago, Columbia, Maryland, Ohio, Wisconsin, Yale, ··· ···, England, Canada, Japan, Poland, Germany, France, Finland, ······, Ann. of Math., Mathematika, J.Number Theory, Glasgow Math. J., London Math. Soc., In. J. Math. Math. Sci., Acta Arith., Can. Math. Bull. (They refused the publications of my papers), J. reine angew. Math.. Both papers were published in Chinese. FLT is as simple as Pythagorean theorem. This proof can fit in the margin of Fermat book. We think the game is up. We sent dept of math (Princeton University) a preprint on Jan. 15, 1992. Andrew, Wiles claims the second proof of FLT in England (not in U.S.A.) after two years. We wish Andrew, Wiles and his supporters disprove my proof, otherwise Wiles work is only the second and complex proof of FLT. We believe that the Princeton is the fairest University and history will pass the fairest judgment on proofs of FLT and other problems.

We are waiting for word from the experts who are studying this paper.